Facilitating Mathematical Thinking with Effective Questions

Common Core State Standards, Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

- **Make Sense of the Problem** *Can students understand, define, formulate, or explain the problem or task?*
  - What is this problem about?
  - What do you need to find out?
  - What information do you have?
  - Is there something that can be eliminated or that is missing?
  - What assumptions do you have to make?
- **Make Conjectures** *Can students describe the meaning of the solution and plan a solution pathway?*
  - What would be a good (estimate/prediction)?
  - What are some possible…(next steps/strategies/solutions)?
  - What patterns do you see?
  - What would happen if…? What if not?
  - What might be an easier problem you could start with?
- **Monitor and Evaluate Progress** *Can students vary the approach if one approach is not working?*
  - Would another method work as well or better?
  - Is there another way to (draw, explain, say…) that?
  - What did not work? Why?
  - Have you tried…(tables, lists, diagrams, manipulatives…)?
  - Would it help to draw a diagram or make a sketch?
- **Assess Solutions** *Do students evaluate the accuracy and meaning of their solution?*
  - Other than retracing your steps, how can you determine if your answer is correct?
  - Is the solution reasonable, considering the context?
  - Is that the only possible solution?
  - How do you know if you are finished?
  - Does that make sense?

2. Reason abstractly and quantitatively.

- **Make Sense of Quantities and Relationships** *Do students see relationships and recognize central ideas?*
  - What does this number represent?
  - What units are you working with?
  - What is the relationship between…?
- **Represent the Problem Symbolically** *Can students decontextualize and contextualize the quantities in relation to the problem situation?*
  - How can you represent this a different way?
  - What does this (symbol, diagram,…) represent?

3. Construct viable arguments and critique the reasoning of others.

- **Make Conjectures** *Do students use what they know to construct arguments?*
  - What you know about ___? How can you use that information?
  - What assumptions are you making?
  - Can you think of a counterexample?
  - Will that be true for all cases?
- **Justify Conclusions** *Can students articulate their thought processes? Do they explain their reasoning?*
  - How did you reach that conclusion?
  - How did you get your answer?
  - Why does that make sense?
- **Critique the Reasoning of Others** *Can students make sense of someone else's reasoning? Can they recognize correct reasoning from that which is flawed?*
  - What do you think about what ___ said?
  - Do you agree? Why or why not?
  - What questions does this raise for you?
  - What could you add to improve this line of reasoning?
  - Which is the most efficient method? Why?

National Council of Teachers of Mathematics (1991)
Adapted from *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions*

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4. Model with mathematics.

- **Apply the Mathematics** *Do students use what they know to solve problems arising in everyday life, society, and the workplace?*
  - How can you apply what you know to this problem?
  - What is the relationship between ___? How could you model this using…(a diagram, tables, graph, flowchart, equation)?
  - How could you (organize/represent) these ideas?
  - If this is not working for you, how might you adjust the (diagram, table, graph, equation,..) to represent the situation better?
- **Interpret Results** *Do students interpret their results in the context of the situation?*
  - Does that make sense in terms of the situation?
  - Does your answer need to be adjusted to fit the situation?

5. Use appropriate tools strategically.

- **Choose Appropriate Tools** *Can students use a variety of tools (paper and pencil, concrete models, rulers, protractors, compasses, calculators, and other technology) to solve problems?*
  - What could you use to help you ….?
  - What would be the (quickest/easiest/most efficient) way to …?
  - How could you use (calculator/computer) to…?
  - What are some (benefits/limitations) of (using/not using)…?

6. Attend to precision.

- **Communicate Precisely** *Do students use appropriate terminology and notation?*
  - How would you say that using appropriate math vocabulary?
  - What does that (symbol/variable/axis/number) represent?
  - Did you define your variables?
  - Did you label your (answer/diagram/graph) appropriately?
- **Express Answers Precisely** *Do students express numerical answers with a degree of precision appropriate for the problem context?*
  - Would it make sense to round your answer? To what place value? Why?
  - How exact does your answer need to be? (in terms of units or place value)

7. Look for and make use of structure.

- **Look for Structure** *Do students look closely to discern a pattern or structure?*
  - What do you notice about …?
  - How is this similar to …?
  - What are two different ways we could (look at/describe/write) that?
- **Make Use of Structure** *Do students use patterns or structures they know to solve problems a different way or more efficiently?*
  - How could you use that (pattern/structure) to …?
  - What do you know about the value of this expression?

8. Look for and express regularity in repeated reasoning.

- **Look for Repeated Reasoning** *Can students recognize when calculations are repeated?*
  - What do you notice about the last few…?
  - How are these similar to each other?
- **Express Regularity** *Can students generalize methods?*
  - How could you describe that pattern in words?
  - How could you describe that pattern as a rule?

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