Formative Assessment in Mathematics: Responding to Students in Ways that Improve Their Learning

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February 22, 2016
Lessons Learned from the World’s Best Performing School Systems

McKinsey and Co. 2008

The quality of a system cannot exceed the quality of its teachers.
The only way to improve outcomes is to improve instruction.
High performance requires every child to succeed.
Great leadership at the school level is a key enabling factor.
How to Improve Math Teaching

Enhance Teacher Knowledge
Raise the Cognitive Demand
Promote Student Discourse
Address the Issues of Status

David Foster
Good Instruction Makes A Difference

Good teaching can make a significant difference in student achievement, equal to one effect size (a standard deviation), which is also equivalent to the affect that demographic classifications can have on achievement.

Paraphrase Dr. Heather Hill, University of Michigan
“What Matters Very Much is Which Classroom”

If a student is in one of the most effective classrooms he or she will learn in 6 months what those in an average classroom will take a year to learn. And if a student is in one of the least effective classrooms in that school, the same amount of learning take 2 years.

*Most effective classes learn 4 times the speed of least effective.*

Dylan Wiliams, University of London
How can problems be used to assess performance?

How can this assessment be used to promote learning?

What kinds of feedback are most helpful for students and which are unhelpful?

How can students become engaged in the assessment process?
Why do you assess students? What different purposes do your assessments serve?

- to diagnose difficulties
- to celebrate achievement
- to motivate students
- to select students for classes
- to maintain records
- to keep teachers and parents informed of progress
- to assess teaching methods.
There are two main purposes of assessment

*Summative assessment* – to summarize and record overall achievement at the end of a course, for promotion and certification. Most ‘high stakes’ tests and external examinations are designed for this purpose. Summative assessment is also used to evaluate the relative effectiveness of a particular course, teaching method, or even an institution.
There are two main purposes of assessment

*Formative assessment* – to recognize achievements and difficulties at the beginning or during a course, so that teachers and students can take appropriate action. This type of assessment forms an integral part of all learning.
Formative Assessment

... all those activities undertaken by teachers, and by their students in assessing themselves, which provide information to be used as feedback to modify the teaching and learning activities in which they are engaged. Such assessment becomes ‘formative assessment’ when the evidence is actually used to adapt the teaching work to meet the needs. (Black & Wiliam, 1998)
Formative Assessment

Students and teachers using evidence of learning to adapt teaching and learning to meet immediate learning needs minute-to-minute and day to day

David Foster
Fractions

This problem gives you the chance to:
- show the position of fractions on a number line
- compare the sizes of fractions

Here is a number line.

0  \[\frac{1}{2}\]  1

1. Mark the position of the two fractions \(\frac{2}{3}\) and \(\frac{2}{5}\) on the number line.

2. Explain how you decided where to place \(\frac{2}{3}\) and \(\frac{2}{5}\) on the number line.

   ____________________________________________
   ____________________________________________
   ____________________________________________
   ____________________________________________

3. Which of the two fractions, \(\frac{2}{3}\) or \(\frac{2}{5}\), is nearer to \(\frac{1}{2}\)? _____________

   Explain how you figured it out.
Performance Assessment Task
Fractions Grade 5

This task challenges a student to use knowledge of fractions and place value system to locate numbers on a number line. A student must use understanding of fractions, their equivalents, and decimals and related between these representations to order and compare values and round to a given value. A student must be able to construct a justification for comparing fractions.
Common Core State Standards Math Content Standards

Number and Operations in Base Ten

Understand the place value system.

5.NBT.3 Read, write, and compare decimals to thousandths.

b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

5.NBT.4 Use place value understanding to round decimals to any place.
Number and Operations – Fractions
Use equivalent fractions as a strategy to add and subtract fractions.

5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole including cases of unlike denominators, e.g., by using visual fractions models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2/5 + 1/2 = 3/7$. by observing that $3/7 < 1/2$. 
Common Core State Standards Math Content Standards

**Number and Operations – Fractions**

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem, *For example, interpret \( \frac{3}{4} \) as the result of dividing 3 by 4, noting that \( \frac{3}{4} \) multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size \( \frac{3}{4} \). If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between that two what two whole numbers does your answer lie?*
Common Core State Standards Math Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

MP.5 Use appropriate tools strategically.
# Fractions

The core elements of performance required by this task are:
- show the position of fractions on a number line
- compare the sizes of fractions

Based on these, credit for specific aspects of performance should be assigned as follows

<table>
<thead>
<tr>
<th></th>
<th>Fractions correctly marked on the number line:</th>
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<tbody>
<tr>
<td>1</td>
<td>$\frac{2}{5}$ to the left of $\frac{1}{2}$</td>
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<tr>
<td></td>
<td>$\frac{2}{3}$ to the right of $\frac{1}{2}$</td>
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<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>1</th>
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<tbody>
<tr>
<td></td>
<td>2</td>
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<table>
<thead>
<tr>
<th></th>
<th>Gives correct explanation such as:</th>
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<tbody>
<tr>
<td>2</td>
<td>$\frac{2}{5}$ is less than $\frac{1}{2}$ and $\frac{2}{3}$ is more than $\frac{1}{2}$</td>
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<td></td>
<td>Accept explanations based on diagrams.</td>
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<thead>
<tr>
<th></th>
<th>Gives correct answer: $\frac{2}{5}$ dependent on some correct explanation/work</th>
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<tbody>
<tr>
<td>3</td>
<td>Shows work such as:</td>
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<tr>
<td></td>
<td>$\frac{2}{3} = \frac{20}{30}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{2}{5} = \frac{12}{30}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{2} = \frac{15}{30}$</td>
</tr>
<tr>
<td></td>
<td>so $\frac{2}{5}$ is nearer to $\frac{1}{2}$</td>
</tr>
</tbody>
</table>

**or**

Accept diagrams showing the line divided into 5 equal parts, and three equal parts, with $\frac{2}{3}$ and $\frac{2}{5}$ correctly marked.

**Partial credit**

Correct reasoning with arithmetical errors.

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<th>3</th>
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</table>

**Total Points** 6
Here is a number line.

1. Mark the position of the two fractions \( \frac{2}{3} \) and \( \frac{2}{5} \) on the number line.

2. Explain how you decided where to place \( \frac{2}{3} \) and \( \frac{2}{5} \) on the number line.

   I made all the fractions to have the same denominator and I put the fractions either before \( \frac{1}{2} \) or beyond \( \frac{1}{2} \). I got \( \frac{12}{20} \) for \( \frac{2}{3} \), \( \frac{28}{30} \) for \( \frac{2}{5} \), and \( \frac{15}{30} \) for \( \frac{1}{2} \). I compared those two numbers with \( \frac{1}{2} \) or \( \frac{2}{3} \).

3. Which of the two fractions, \( \frac{2}{3} \) or \( \frac{2}{5} \), is nearer to \( \frac{1}{2} \)? \( \frac{2}{3} \) is closer. \( \frac{2}{5} \) is further.

Explain how you figured it out.

I made the fractions to have the same denominator and I compared them. \( \frac{2}{3} \) was \( \frac{2}{6} \) off of \( \frac{1}{2} \). \( \frac{2}{5} \) was \( \frac{1}{10} \) off of \( \frac{1}{2} \). So \( \frac{2}{3} \) is closer.
Here is a number line.

1. Mark the position of the two fractions \( \frac{2}{3} \) and \( \frac{2}{5} \) on the number line.

2. Explain how you decided where to place \( \frac{2}{3} \) and \( \frac{2}{5} \) on the number line.

   Because \( \frac{2}{3} \) is 60% and 60% is close to \( \frac{1}{2} \), so we estimated where both are. \( \frac{2}{3} \) is closer so we got 60%.

3. Which of the two fractions, \( \frac{2}{3} \) or \( \frac{2}{5} \), is nearer to \( \frac{1}{2} \)?

   Explain how you figured it out.

   Because \( \frac{1}{2} \) is 50% and \( \frac{2}{3} \) is 66% and \( \frac{2}{5} \) is 40% so 40% (\( \frac{2}{5} \)).
Here is a number line.

1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.
   
   I divided the top part of the line into thirds and marked $\frac{2}{3}$ and divided the bottom part of the line into fifths and marked $\frac{2}{5}$.

3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$?

   Explain how you figured it out.

   On the number line the dash that marks $\frac{2}{5}$ is further from $\frac{1}{2}$ than the dash that marks $\frac{2}{3}$.
Comparing Fractions (A)

Compare each pair of fractions using a <, > or = sign.

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<tbody>
<tr>
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<td>2/4</td>
<td>15/3</td>
<td>7/4</td>
<td>6/4</td>
<td>2/3</td>
<td>1/4</td>
<td>1/3</td>
<td></td>
</tr>
<tr>
<td>1/3</td>
<td>16/4</td>
<td>2/4</td>
<td>3/4</td>
<td>4/3</td>
<td>10/5</td>
<td>2/3</td>
<td>2/3</td>
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<tr>
<td>2/3</td>
<td>4/2</td>
<td>6/6</td>
<td>2/3</td>
<td>1/5</td>
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<td>16/6</td>
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<tr>
<td>5/2</td>
<td>1/2</td>
<td>4/5</td>
<td>1/2</td>
<td>6/5</td>
<td>16/6</td>
<td>10/4</td>
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<tr>
<td>16/4</td>
<td>4/6</td>
<td>3/6</td>
<td>2/5</td>
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<td>3/5</td>
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<td>17/5</td>
<td>1/3</td>
<td>14/4</td>
<td>2/5</td>
<td>1/2</td>
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<tr>
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<td>1/2</td>
<td>2/6</td>
<td>1/2</td>
<td>1/2</td>
<td>8/2</td>
<td>1/5</td>
<td>1/2</td>
<td></td>
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</tbody>
</table>
What kinds of feedback are most helpful for students and which are unhelpful?
Traditionally teachers choose one of three options:

• Go back and reteach the topic with the entire class
• Identify the students needing remediation and find some time/opportunity to reteach the topic while the rest of the class continues on.
• Feeling the pressure of the over packed curriculum the teacher ventures on to the next topic.
The dangers of giving marks, levels, rewards and rankings

Where the classroom culture focuses on rewards, ‘gold stars’, grades or place-in-the-class ranking, then pupils look for the ways to obtain the best marks rather than at the needs of their learning which these marks ought to reflect. One reported consequence is that where they have any choice, pupils avoid difficult tasks. They also spend time and energy looking for clues to the ‘right answer’. Many are reluctant to ask questions out of fear of failure. Pupils who encounter difficulties and poor results are led to believe that they lack ability, and this belief leads them to attribute their difficulties to a defect in themselves about which they cannot do a great deal. So they ‘retire hurt’, avoid investing effort in learning which could only lead to disappointment, and try to build up their self-esteem in other ways. Whilst the high-achievers can do well in such a culture, the overall result is to enhance the frequency and the extent of under-achievement.
• What are the implications of this for classroom practice?
• What would happen if we stopped giving marks or levels on pupils’ work?
• Why are so many teachers resistant to making this change?
The advantages of giving clear, specific, content-focused feedback

What is needed is a culture of success, backed by a belief that all can achieve. Formative assessment can be a powerful weapon here if it is communicated in the right way. Whilst it can help all pupils, it gives particularly good results with low achievers where it concentrates on specific problems with their work, and gives them both a clear understanding of what is wrong and achievable targets for putting it right. Pupils can accept and work with such messages, provided that they are not clouded by overtones about ability, competition and comparison with others. In summary, the message can be stated as follows:

Feedback to any pupil should be about the particular qualities of his or her work, with advice on what he or she can do to improve, and should avoid comparisons with other pupils.
• What are the implications of this for classroom practice?
• Does this kind of feedback necessarily take much longer to give?
Looking at Student Work

The process of studying student work is a meaningful and challenging way to be data-driven, to reflect critically on our instructional practices, and to identify the research we might study to help us think more deeply and carefully about the challenges our students provide us. Rich, complex work samples show us how students are thinking, the fullness of their factual knowledge, the connections they are making. Talking about them together in an accountable way helps us to learn how to adjust instruction to meet the needs of our students.

Annenberg Institute of School Reform
Educational Research:
Formative Assessment & Student Work to Inform Instruction

Assessing Student Outcomes; Marzano, Pickering, McTighe
Inside the Black Box; Black, Williams
Understanding by Design; Wiggins, McTighe
Results Now; Schmoker
Professional Learning Communities at Work; Dufour, Eaker
Accountability for Learning; Reeves
Math Talk Learning Community; Fuson, et al
Normalizing Problems of Practice; Little, Horn
Change the Terms for Teacher Learning; Fullan
Working toward a continuum of professional development; Loucks-Horsley, et al.


**Fractions** – 2005, performance task by Mathematics Assessment Resource Service (MARS) available with permission, page 8


**Formative Assessment**, MARS, Shell Centre, University of Nottingham 2012